Correspondence

A Mode Chart for Accurate Design of Cylindrical Dielectric Resonators

Awareness of dielectric resonators as useful microwave circuitry elements is indicated by a number of recent publications [2]–[6]. Using intermediate and high dielectric constant materials of low loss, the resonators can provide a convenient means of miniaturization and field concentration. The work reported here was motivated by the need for resonators having dielectric constants of 13 to 16 to provide field concentration in ferrite crystals at 74 Gc.

Solution of the dispersive characteristics derived from transmission structure analyses, is usually displayed in the form [1]

$$\frac{\lambda}{\lambda_0} = \sqrt{\frac{V^2 + W^2}{V^2 + \epsilon_r W^2}} \qquad (1)$$

$$\frac{D}{\lambda_0} = \frac{1}{\pi}\sqrt{\frac{V^2 + W^2}{\epsilon_r - 1}} \qquad (2)$$

where λ and λ_0 are wavelengths in the structure and in free space, V and W are the internal and external separation constants, and D is the diameter of the cylinder. Dispersive data of this form for a wide range of dielectric constants for the TE_{01} and TM_{01} modes are presented by Longaker and Roberts [4]. Figure 1 gives data for the HE_{11} mode, which is the dominant mode of the rod.

For many resonator applications, presentation in a form analogous to the mode chart representation for metallic cavities is more convenient [6].

Equations (1) and (2) may be combined into the form

$$(fD)^2 = \left(c\frac{l}{2}\right)^2 \left(\frac{V^2 + W^2}{V^2 + \epsilon_r W^2}\right) \left(\frac{D}{L}\right)^2 \qquad (3)$$

where D and L are the diameter and length, respectively, c is the free space propagation velocity, and l is the resonator mode index in the axial direction. Note that unlike the metallic cavity [7], the $(fD)^2$ vs. $(D/L)^2$ characteristics for dielectric resonators are not straight lines but asymptote to different values at large and small diameters. For diameters sufficiently close to cutoff $V \gg W$, the slope reduces to $c^2l^2/4$. For sufficiently large diameters, $W \gg V$, the slope becomes $c^2l^2/4\epsilon_r$. Also in contrast to the metallic cavities, the $(fD)^2$ vs. $(D/L)^2$ dialectric characteristics do not in general terminate at the left-hand side but at the points

$$(fD)^2 = \frac{c^2 V^2_{nm}}{\pi^2(\epsilon_r - 1)};$$

$$\left(\frac{D}{L}\right)^2 = \frac{V^2_{nm}}{\left(\frac{l\pi}{2}\right)^2 (\epsilon_r - 1)}$$

where V_{nm} is the cutoff value of V for the mode having circumferential and radial indices n and m. For $n = 0$, 1, V is equal to j_{nm}, the mth zero of the nth order Bessel functions of the first kind. A mode chart of the lower order modes for $\epsilon_r = 15$, 16 is shown in Fig. 2.

Advantages of this chart over presentations such as shown in Fig. 1 include: 1) for design purposes, and display of the modes on a geometrical basis, clearly indicating areas of light mode density, and 2) for reduction of experimental data, a unique representation for each measured resonance. Mode order

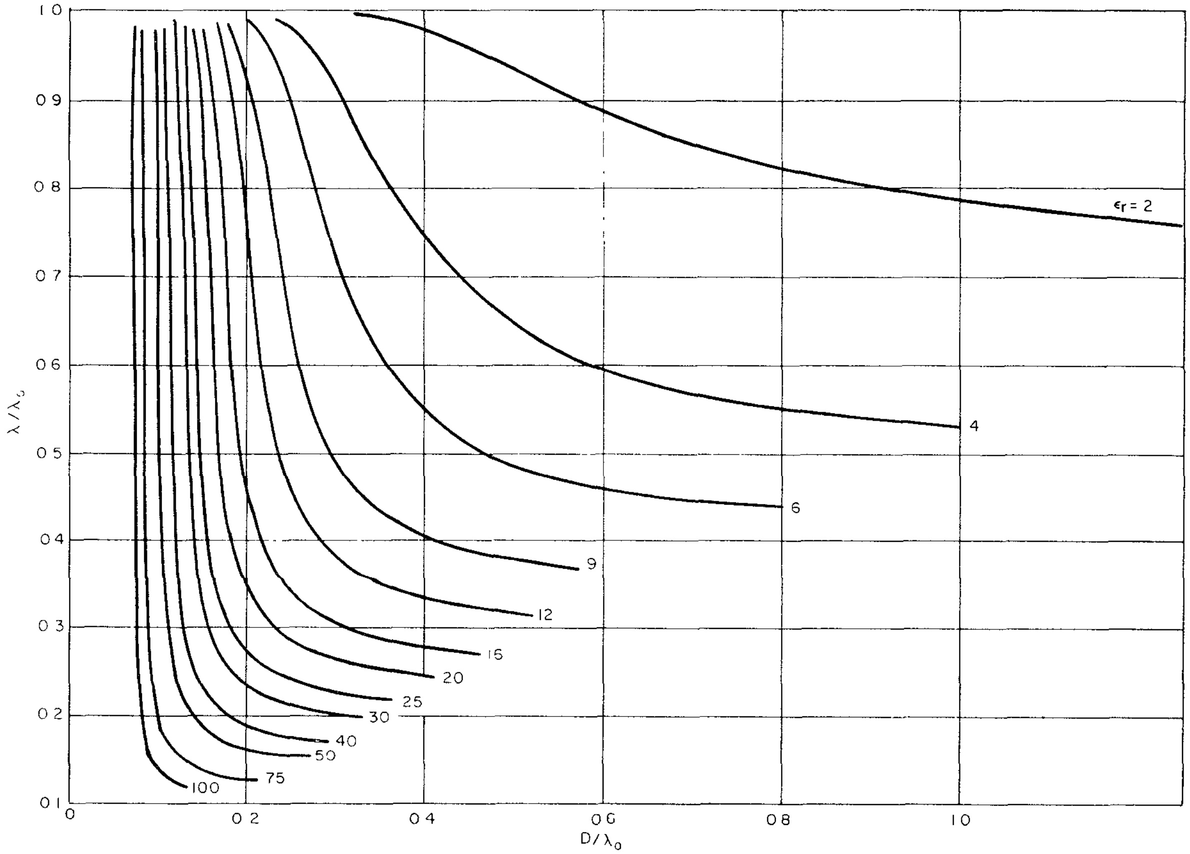

Fig. 1. HE_{11} mode dielectric rod dispersive characteristics.

Manuscript received March 1, 1965. This work was supported in part by the United States Army Signal Corps under Contract DA-36-039-SC-89071.

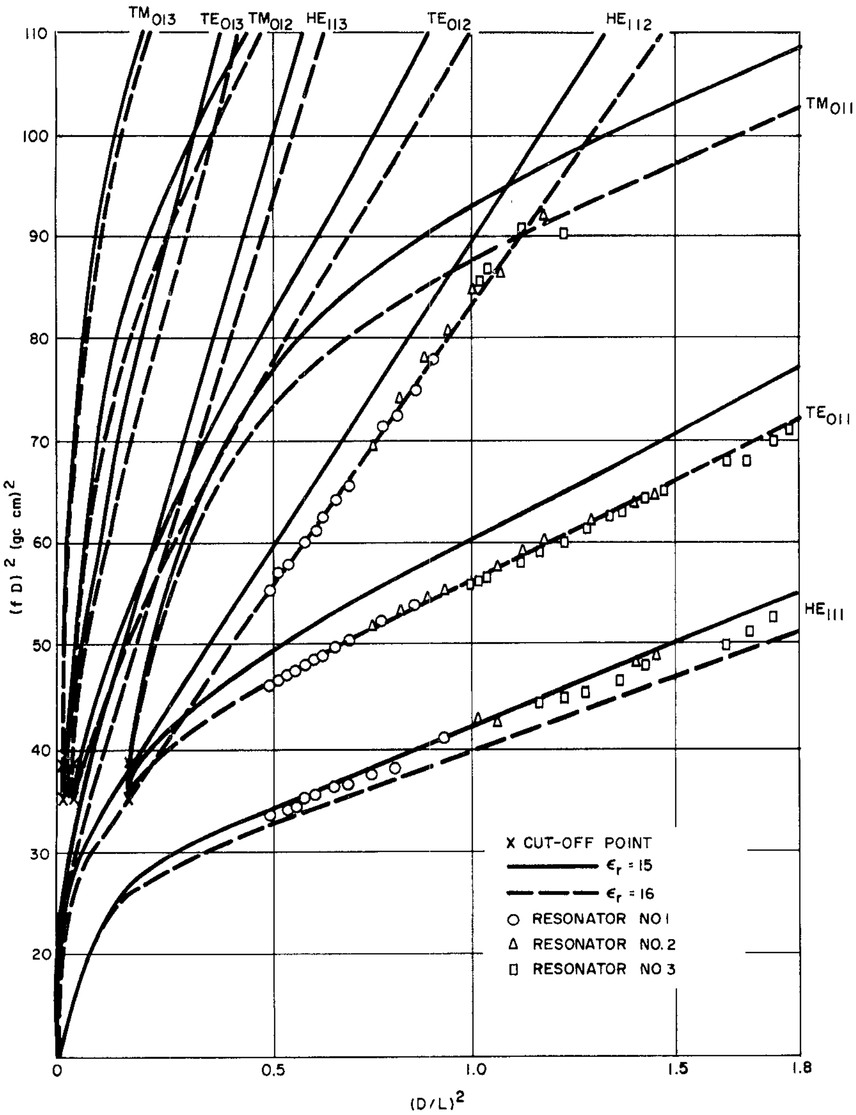

Fig. 2. Mode chart with experimental points for $MgTiO_3$.

must be determined for plotting experimental points on the dispersive characteristics shown in Fig. 1. For some experimental situations the unique representation of the mode chart is a significant advantage as less prior knowledge of the dielectric constant is needed to make reliable identification.

The experimental points were taken on resonators constructed from a magnesium titanate ceramic for which the nominal dielectric constant is 16 [8]. Data taken for each resonator are indicated by the circle, triangle or square. Successive points for each resonator were obtained by incrementally grinding the length and measuring the resonant frequency at each step. The mount used for these measurements closely resembles the parallel plate arrangement used by Hakki and Coleman [2] to measure the properties of TE modes from lower dielectric constant materials. Excitation was similarly obtained by means of an iris in one of the plates.

The most consistent results were obtained on the TE_{011} mode. This is in agreement with the suggested use of TE modes by Hakki and Coleman. It is also in agreement with the suggestion of Cohn and Kelly [6] that greatest measurement accuracy can be obtained on modes for which no electric field component exists normal to the dielectric surface. This avoids the effect of an apparent series capacity in the air gaps between the dielectric and conducting plates. The high degree of consistency for successive points for the TE_{011} mode, less than ± 0.2 per cent from a best fit curve for each resonator, suggests that a major source of the deviations was in the fabrication procedure.

Although light coupling is desirable for dielectric constant measurements, many applications require tight coupling to the resonators. On the present program, resonators have been constructed for exciting ferrite crystals from X through M band frequencies. No difficulty was encountered in achieving critical coupling in the lower order modes. Critical coupling for 0.300-inch diameter, 0.22- to 0.4-inch length posts at X band was accomplished through a 0.140-inch iris in a 0.010-inch wall.

In summary, the mode chart has been shown to be a very convenient means of mode selection and display for dielectric resonators. When used in conjunction with TE_{011} dispersive measurements, it affords a simple, accurate method of determining the dielectric constant for a wide range of dielectric values.

ACKNOWLEDGMENT

The authors would like to thank B. R. Hubble for fabrication of the ceramic resonators and aid in development of the related measurement equipment.

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Rectangular Waveguide Flange Nomenclature

The well-known waveguide nomenclature muddle is exceeded in lack of clarity by the flange nomenclature muddle. For every rectangular waveguide size, which may have several different designations, there are several flanges available, each with its own array of designations. The flange nomenclature muddle may become a major annoyance to those users who use equipment made by different manufacturers in different countries, for manufacturers often state the flange numbers of a device without stating the waveguide size or flange nomenclature system used.

Manuscript received February 12, 1965.